

**TOR TEBRANISH TENGLAMASI UCHUN CHEGARALANMAGAN
SOHADA BOSHLANG'ICH – CHEGARAVIY MASALANI FURE
ALMASHTIRISHI BILAN YECHISH**

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Annotatsiya Ushbu maqolada tor tebranish tenglamasiga qo'yilgan boshlang'ich-chegaraviy masalani yechish tadqiq qilinadi. Mazkur masalani Furyening integral almashtirishi usuli bilan yechish masalasi ko'rib chiqiladi. Bunda qaralayotgan soha chegaralanmagan soha bo'lib, integral almashtirish formulasi yordamida masala oddiy differensial tenglamaga keltiriladi. Umumiy yechim Furyening teskari almashtirishiga ko'ra topiladi.

Kalit so'zlar: Furyening integral almashtirishlari, chegaralanmagan soha, oddiy differensial tenglama, boshlang'ich-chegaraviy masala, umumiy yechim.

Аннотация В данной статье исследуется задача Коши для уравнения колебаний струны. Рассматривается решение этой задачи с использованием метода интегрального преобразования Фурье. Область, в которой рассматривается задача, является неограниченной. С помощью формулы интегрального преобразования задача сводится к обыкновенному дифференциальному уравнению. Общее решение находится с применением обратного преобразования Фурье.

Ключевые слова: интегральные преобразования Фурье, неограниченная область, обыкновенное дифференциальное уравнение, начально-краевая задача, общее решение

Annotation This article investigates the Cauchy problem for the string vibration equation. The solution to this problem is studied using the Fourier integral

transform method. The domain under consideration is unbounded, and by applying the Fourier transform formula, the problem is reduced to an ordinary differential equation. The general solution is obtained using the inverse Fourier transform.

Keywords: Fourier integral transforms, unbounded domain, ordinary differential equation, initial-boundary value problem, general solution

Kirish

Masala. $\Omega = \{(x, t) : 0 < x < +\infty, 0 < t < +\infty\}$ sohada

$$U_{tt} = a^2 U_{xx} \quad (1)$$

tenglamaning quyidagi

$$U_x(0, t) = 0 \quad 0 \leq t < +\infty \quad (2)$$

$$U(x, 0) = f(x) \quad U_t(x, 0) = g(x), \quad 0 \leq x \leq +\infty \quad (3)$$

shartlarni qanoatlantiruvchi yechimi topilsin.

Masalaning yechimini Furening integral almashtirishlari usuli yordamida topamiz. Masalaga qo'yilgan chegaraviy shartga ko'ra Furening kosinus almashtirishidan foydalanamiz. Umumiy yechim $U(x, t)$ ni mazkur almashtirish orqali yozib olamiz. Bunda, bizga berilgan to'liq tenglamasi oddiy differensial tenglama ko'rinishiga keladi va buning umumiy yechimini topish mumkin bo'ladi:

$$U(x, t) = \sqrt{\frac{2}{\pi}} \int_0^{\infty} U_0(\lambda, t) \cos \lambda x d\lambda \quad (4)$$

Yuqoridagi ifodadan avval kerakli tartibda hosilalarini topamiz hamda berilgan shartlarga ko'ra yozib olamiz:

$$U_x(x, t) = -\sqrt{\frac{2}{\pi}} \lambda \int_0^{\infty} U_0(\lambda, t) \sin \lambda x d\lambda \quad (5)$$

Bundan, $U_x(0, t) = 0$.

(5) tenglikdan ikkinchi marta hosila olamiz :

$$U_{xx}(x, t) = -\sqrt{\frac{2}{\pi}} \lambda^2 \int_0^{\infty} U_0(\lambda, t) \cos \lambda x d\lambda$$

Xuddi shu kabi t argument bo'yicha birinchi va ikkinchi tartibli hosilalarini yozamiz :

$$U_t(x, t) = \sqrt{\frac{2}{\pi}} \int_0^{\infty} U_t''(\lambda, t) \cos \lambda x d\lambda$$

va

$$U_{tt}(x, t) = \sqrt{\frac{2}{\pi}} \int_0^{\infty} U_{tt}''(\lambda, t) \cos \lambda x d\lambda.$$

Yuqoridagi ifodalardan tenglamaning umumiy yechimini Fure almashtirishidan hosil bo'lgan oddiy differensial tenglamaga ko'rinishiga olib kelamiz:

$$\sqrt{\frac{2}{\pi}} \int_0^{\infty} U_{tt}''(\lambda, t) \cos \lambda x d\lambda = -\sqrt{\frac{2}{\pi}} \lambda^2 a^2 \int_0^{\infty} U''(\lambda, t) \cos \lambda x d\lambda$$

Bundan,

$$U_{tt}''(\lambda, t) = -\lambda^2 a^2 U''(\lambda, t)$$

Chegaraviy shartlarni quyidagicha hosil qilamiz:

$$U(x, 0) = \sqrt{\frac{2}{\pi}} \int_0^{\infty} U''(\lambda, 0) \cos \lambda x d\lambda$$

ga ko'ra ,

$$U_t(x, 0) = \sqrt{\frac{2}{\pi}} \int_0^{\infty} U_t''(\lambda, 0) \cos \lambda x d\lambda$$

ni yozish mumkin.

Berilganlarga ko'ra ular mos ravishda

$$f(x) = \sqrt{\frac{2}{\pi}} \int_0^{\infty} U''(\lambda, 0) \cos \lambda x d\lambda$$

va

$$g(x) = \sqrt{\frac{2}{\pi}} \int_0^{\infty} U_t''(\lambda, 0) \cos \lambda x d\lambda$$

ga teng bo'ladi. Furening teskari almashtirishiga binoan :

$$U(\lambda, 0) = \sqrt{\frac{2}{\pi}} \int_0^{\infty} f(x) \cos \lambda x d\lambda; \quad U_t(\lambda, 0) = f(\lambda)$$

$$U_t(\lambda, 0) = \sqrt{\frac{2}{\pi}} \int_0^{\infty} g(x) \cos \lambda x d\lambda; \quad U_{tt}(\lambda, 0) = g(\lambda)$$

Endi tenglamaning umumiy ko‘rinishini quyidagicha yozish mumkin:

$$\begin{cases} U_{tt}(\lambda, t) + \lambda^2 a^2 U(\lambda, t) = 0 \\ U(\lambda, 0) = f(\lambda), U_t(\lambda, 0) = g(\lambda) \end{cases}$$

Berilgan tenglamani yechish uchun uning xarakteristik tenglamasini tuzamiz:

$$U(\lambda, t) = e^{kx}; \quad U_t(\lambda, t) = ke^{kx}; \quad U_{tt}(\lambda, t) = k^2 e^{kx}$$

$$k^2 e^{kx} + \lambda^2 a^2 e^{kx} = 0; \quad k^2 + \lambda^2 a^2 = 0; \quad k_{1,2} = \pm i$$

Umumiy yechim ko‘rinishini topamiz:

$$U(\lambda, t) = C_1 \cos a\lambda t + C_2 \sin a\lambda t$$

Chegaraviy shartlardan, C_1 va C_2 ni topamiz:

$$C_1 = f(\lambda); \quad C_2 = \frac{g(\lambda)}{a\lambda}$$

Bundan,

$$U(\lambda, t) = f(\lambda) \cos a\lambda t + \frac{g(\lambda)}{a\lambda} \sin a\lambda t$$

Kerakli funksiya $U(x, t)$ ni teskari Fure almashtirishi yordamida topamiz:

$$\begin{aligned} U(x, t) &= \sqrt{\frac{2}{\pi}} \int_0^{\infty} U(\lambda, t) \cos \lambda x d\lambda = \\ &= \sqrt{\frac{2}{\pi}} \int_0^{\infty} \left(f(\lambda) \cos a\lambda t - \frac{g(\lambda)}{a\lambda} \sin a\lambda t \right) \cos \lambda x d\lambda = \\ &= \sqrt{\frac{2}{\pi}} \int_0^{\infty} f(\lambda) \cos a\lambda t \cos \lambda x d\lambda + \sqrt{\frac{2}{\pi}} \int_0^{\infty} g(\lambda) \frac{\sin a\lambda t}{a\lambda} \cos \lambda x d\lambda \end{aligned}$$

Umumiy yechim uchun ikki holatni ko‘rib chiqamiz:

$$1) \quad x - at > 0$$

$$\begin{aligned}
 U(x, t) &= \sqrt{\frac{2}{\pi}} \cdot \frac{1}{2} \int_0^{\infty} f(\lambda) (\cos \lambda(x + at) + \cos \lambda(x - at)) d\lambda + \\
 &+ \frac{1}{2a} \sqrt{\frac{2}{\pi}} \int_0^{\infty} g(\lambda) \frac{(\sin \lambda(x + at) - \sin \lambda(x - at))}{\lambda} d\lambda = \\
 &= \frac{f(x + at) + f(x - at)}{2} + \frac{1}{2a} \sqrt{\frac{2}{\pi}} \left[\int_0^{\infty} g(\lambda) \int_{x-at}^{x+at} \cos(\lambda s) ds \right] d\lambda = \\
 &= \frac{f(x + at) + f(x - at)}{2} + \frac{1}{2a} \int_{x-at}^{x+at} g(s) ds
 \end{aligned}$$

2) $x - at < 0$

$$\begin{aligned}
 U(x, t) &= \sqrt{\frac{2}{\pi}} \cdot \frac{1}{2} \int_0^{\infty} f(\lambda) [\cos \lambda(x + at) + \cos \lambda(at - x)] d\lambda + \\
 &+ \frac{1}{2a} \sqrt{\frac{2}{\pi}} \int_0^{\infty} g(\lambda) \frac{[\sin \lambda(x + at) - \sin \lambda(at - x)]}{\lambda} d\lambda = \\
 &= \frac{f(x + at) + f(at - x)}{2} + \frac{1}{2a} \sqrt{\frac{2}{\pi}} \int_0^{\infty} g(\lambda) \left(\int_{x-at}^{x+at} \cos \lambda s ds \right) d\lambda
 \end{aligned}$$

$g(x)$ funksiya faqat musbat qiymatlardagi x lar uchun aniqlangan. Shuning uchun so'nggi integralni o'zgartiramiz :

$$\begin{aligned}
 \int_{x-at}^{x+at} \cos(\lambda s) ds &= \int_{x-at}^0 \cos(\lambda s) ds + \int_0^{x+at} \cos(\lambda s) ds = \\
 &= - \int_{at-x}^0 \cos(\lambda s) ds + \int_0^{x+at} \cos(\lambda s) ds = \int_0^{at-x} \cos(\lambda s) ds + \int_0^{x+at} \cos(\lambda s) ds
 \end{aligned}$$

Endi:

$$\begin{aligned}
 \sqrt{\frac{2}{\pi}} \int_0^{\infty} g(\lambda) \left(\int_{x-at}^{x+at} \cos(\lambda s) ds \right) d\lambda &= \left[\int_0^{x+at} ds \left(\sqrt{\frac{2}{\pi}} \int_0^{\infty} g(\lambda) \cos(\lambda s) d\lambda \right) + \right. \\
 &+ \left. \int_0^{at-x} ds \left(\sqrt{\frac{2}{\pi}} \int_0^{\infty} g(\lambda) \cos(\lambda s) d\lambda \right) \right] = \int_0^{x+at} g(s) ds + \int_0^{at-x} g(s) ds.
 \end{aligned}$$

Bundan umumiy yechimni quyidagicha bo'ladi:

$$U(x,t) = \frac{f(x-at) + f(x+at)}{2} + \frac{1}{2a} \left[\int_0^{x+at} g(s) ds - \operatorname{sgn}(x-at) \int_0^{|x-at|} g(s) ds \right].$$

FOYDALANILGAN ADABIYOTLAR

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